“I generally hate numbers.”

Translating Instructor Attitudes about Problem-solving Representations to Student Performance in Introductory Mechanics

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SUMMER MEETING
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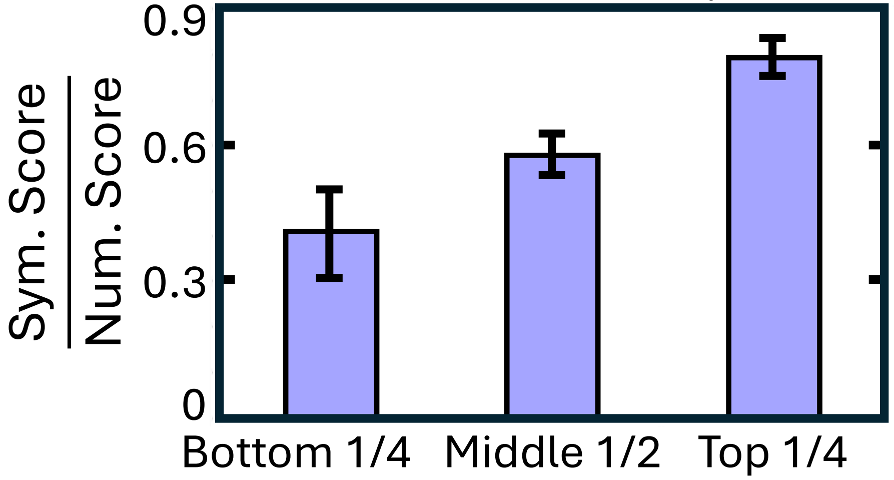
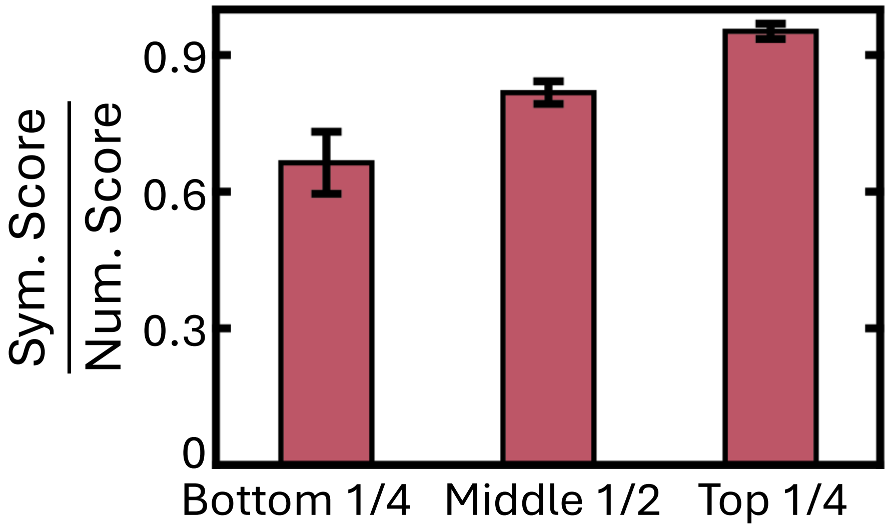
Novice physics students regularly struggle with symbols significantly more than with numbers.

- **Many studies** have shown persistent student **difficulties with symbolic algebra.**
  - **Gaps** between numeric and symbolic proficiency **widen for lower-aptitude** students.
  - Reproduced with open-ended, multipart questions
- Cognitive modeling and student interviews suggest **pattern recognition** as a cause.
- Do the specifics of **student interactions with numbers** on exams affect their scores?
  - Can **instructors** impact these outcomes?

Torigoe & Gladding, *Am J Phys* (2011)



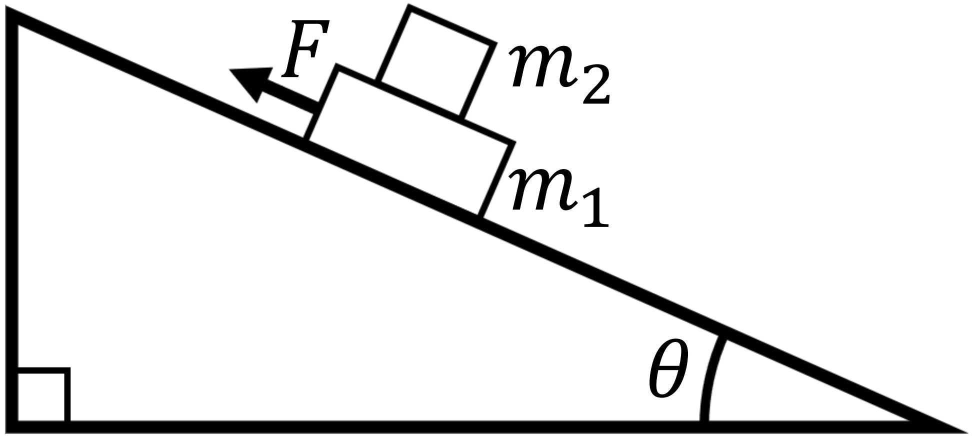
Creyts, Tarr, & Alicea-Muñoz, *AAPT Summer Meet* (2025)



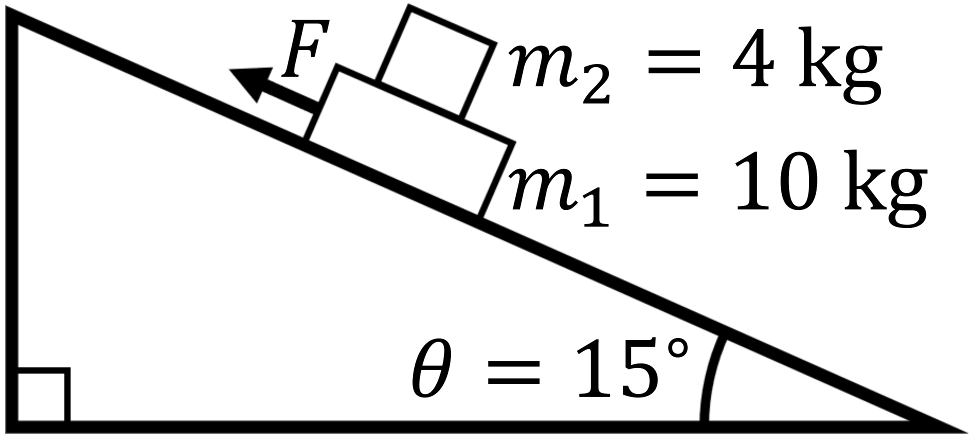
At what stage of numeric problem-solving do students interact with numbers?

- **Manually reviewed** 875 exams and **coded student strategies** not in original rubric for 2 problems
- Equilibrium (part 1):
  - Frictionless ramp
  - Unknown friction between boxes
  - Unknown force on lower box
  - Lower box moves up w/ fixed speed
  - Upper box slides downward
  - **Find μ_k between boxes**

Symbolic



Numeric



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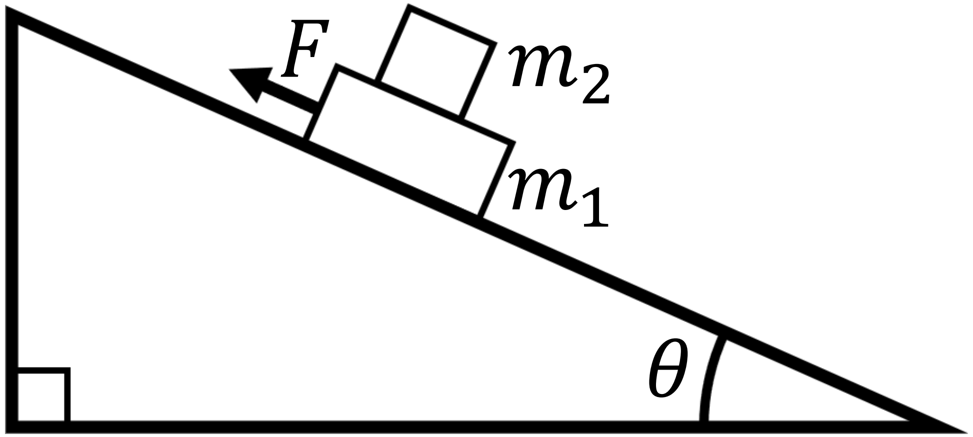

Example solution [Symbolic]

$$F_{net}^{\perp} = 0$$
$$\Rightarrow N_1 - m_2 g \cos\theta = 0$$
$$\Rightarrow N_1 = m_2 g \cos\theta$$

$$F_{net}^{\parallel} = 0$$
$$\Rightarrow m_2 g \sin\theta - \mu_k N_1 = 0$$
$$\Rightarrow m_2 g \sin\theta - \mu_k m_2 g \cos\theta = 0$$
$$\Rightarrow \boxed{\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta}$$

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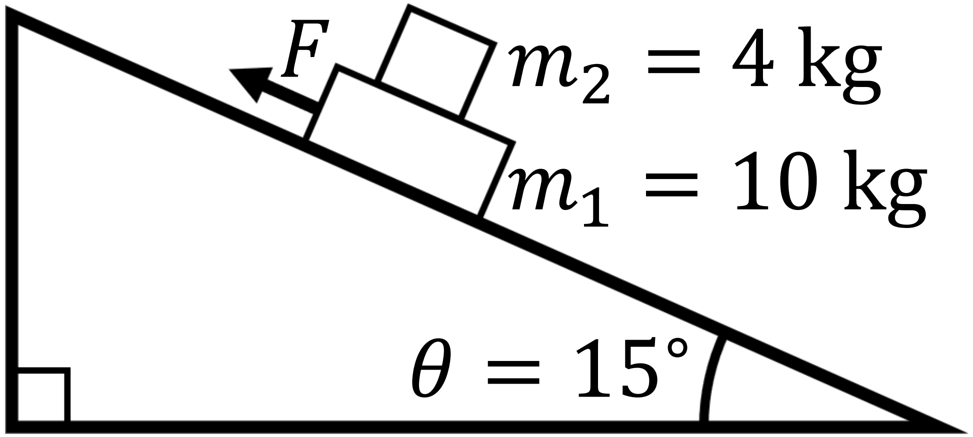

Example solution [Expert-like (Numeric)]

$$F_{net}^{\perp} = 0$$
$$\Rightarrow N_1 - m_2 g \cos\theta = 0$$
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$$\Rightarrow m_2 g \sin\theta - \mu_k m_2 g \cos\theta = 0$$
$$\Rightarrow \mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$
$$\boxed{\mu_k = \tan(15°) \approx 0.268}$$

At what stage of numeric problem-solving do students interact with numbers?

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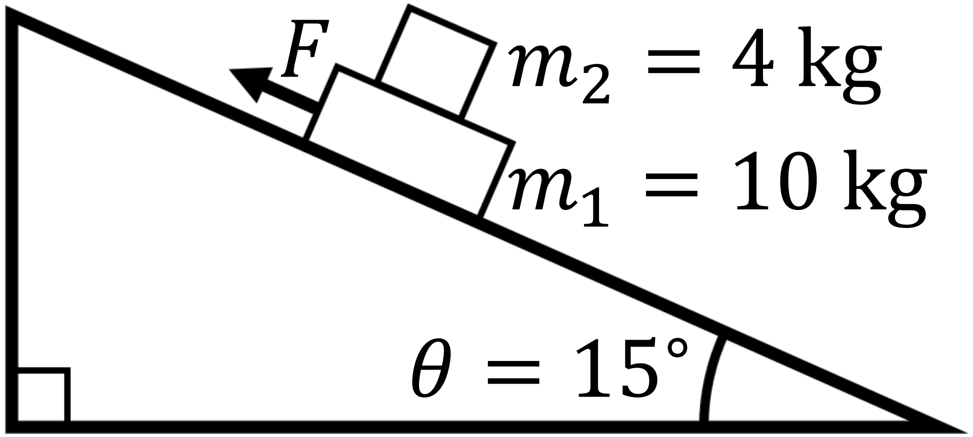

Example solution
[Intermediate (Numeric)]

$$F_{net}^{\perp} = 0$$
$$\Rightarrow N_1 - m_2 g \cos\theta = 0$$
$$\Rightarrow N_1 = m_2 g \cos\theta$$
$$N_1 = (4\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) \cos(15°) \approx 37.86\ \mathrm{N}$$

$$F_{net}^{\parallel} = 0$$
$$\Rightarrow m_2 g \sin\theta - \mu_k N_1 = 0$$
$$\Rightarrow \mu_k = \frac{m_2 g \sin\theta}{N_1}$$
$$\boxed{\mu_k = \frac{(4\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) \sin(15°)}{37.86\ \mathrm{N}} \approx 0.268}$$

At what stage of numeric problem-solving do students interact with numbers?

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- Equilibrium (part 1):
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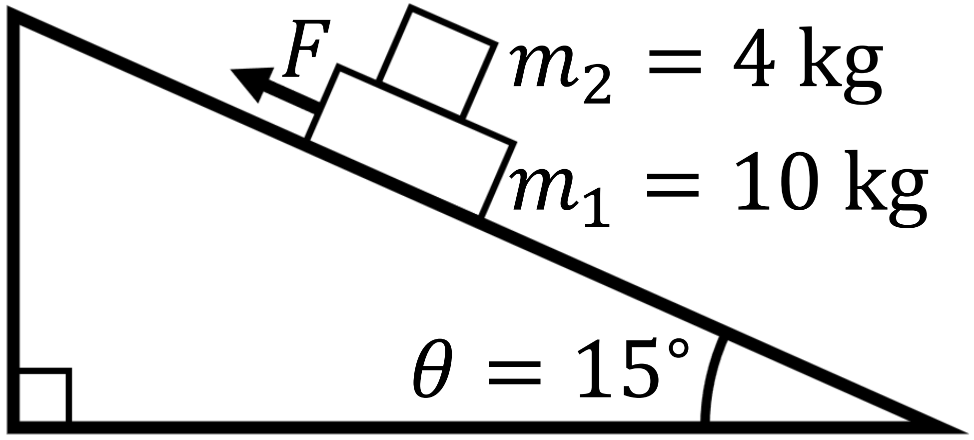
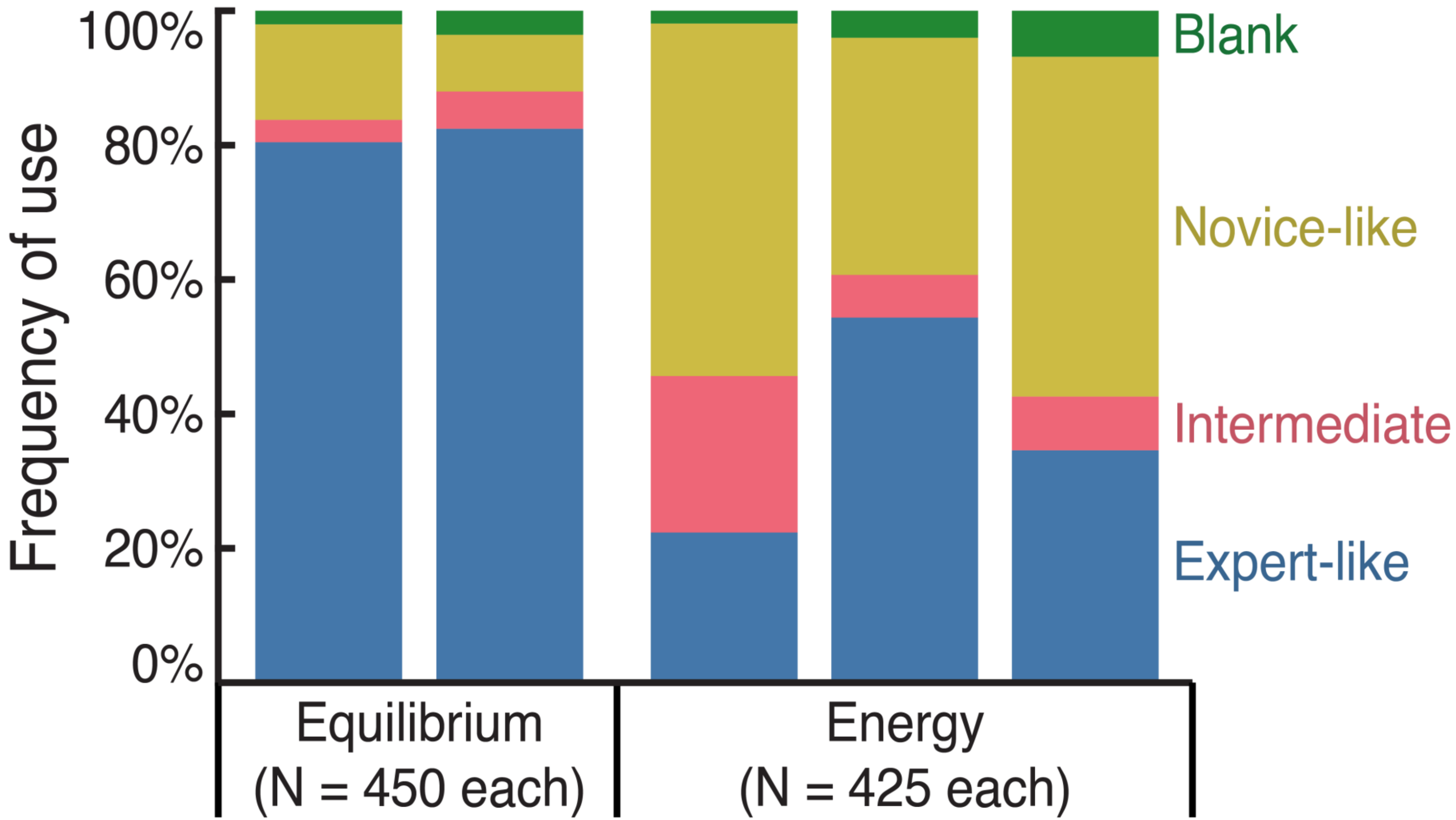


Example solution [Novice-like (Numeric)]

$$F_{net}^{\perp} = 0$$
$$\Rightarrow N_1 - (4\text{ kg})(9.8\text{ m/s}^2)\cos(15°) = 0$$
$$\Rightarrow N_1 \approx 37.86\text{ N}$$

$$F_{net}^{\parallel} = 0$$
$$\Rightarrow m_2 g \sin\theta - \mu_k N_1 = 0$$
$$\Rightarrow (4\text{ kg})(9.8\text{ m/s}^2)\sin(15°) - \mu_k(37.86\text{ N}) = 0$$
$$\boxed{\mu_k = \frac{10.15\text{ N}}{37.86\text{ N}} \approx 0.268}$$

Students engaged in novice-like behaviors on numeric exam problems far less than anecdotal evidence would suggest.



Despite only differing by a final evaluation step, expert-like numeric solutions often scored significantly higher than symbolic solutions.

The presence of numbers alone appears sufficient for slightly but significantly **higher exam performance** in introductory mechanics.

Symbolic never wins!

Pairwise score comparison (adjusted Mann-Whitney *U* test, showing only significant pairs)	How many of 5 parts?	Effect size (Cliff's δ ± SE)
Expert-like > Symbolic	3	0.09 ± 0.02
Intermediate > Symbolic	1	0.08 ± 0.01
Novice-like > Symbolic	1	0.12 ± 0.02
Expert-like > Novice-like	1	0.03 ± 0.01
Intermediate > Expert-like	1	0.011 ± 0.004

The course coordinator provides lecture templates but does not require that instructors follow them.

- Though section instructors cover the **same physics,** they can do so according to their **personal styles and pedagogical philosophies.**

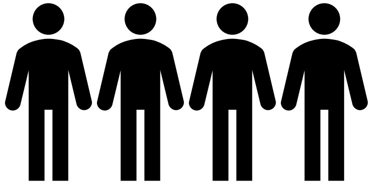
- **What beliefs do** introductory mechanics **instructors** typically **maintain on representations** and student interactions with them?

- **Semi-structured interviews** with 13 instructors who last taught introductory mechanics within 3 years (5 concurrently with interview)
  - Gender distribution: 9 men, 3 women, 1 non-binary
  - Teaching experience: first-timer to 33 years ($M \pm SD = 14 \pm 7$ yr)

Instructors consistently recognized student difficulties with representations but disagreed on how best to address them.

Four emergent philosophies on representations:

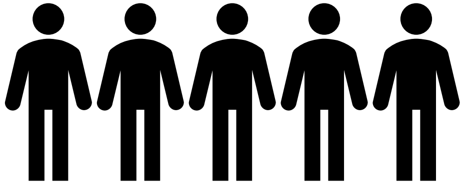
1.



"**Numbers can be a crutch** for the student who is not so adept. **Symbols force them** maybe to **think more carefully** or at least will **expose issues** more readily."

- Heavy **emphasis on symbolic** mastery
- **Numbers used sparingly** and cautiously with **highly specific goals** (usually to simplify unwieldy algebra)

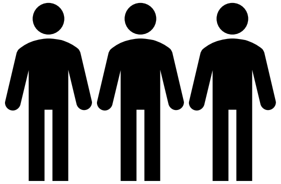
2.



"**Even though it's a numerical problem,** usually when I show them how to do it..., **I really recommend** doing all the steps all the way to the bottom with **symbols**."

- **Balanced exposure** to numbers and symbols...
- ...but **symbolic** mastery remains **underscored during numeric** exercises

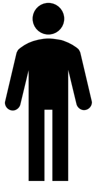
3.



"**They should know both.** They should know how to handle units. They should know how to use expressions. They should know how to do the symbolic algebra. **I don't see that as sort of a big decision.**"

- **Balanced exposure** to numbers and symbols due to **representational agnosticism**
- Focus on universal problem-solving skills and **fluency across all representations**

4.



"It's good for them to make contact with the actual numbers. If you just think about [universal gravitation] from the formula..., what if I have two basketballs? ...**If you don't run the numbers on that, you don't realize** that they basically don't [attract]."

- **Preference for numeric**, real-world understanding
- **Symbols** used for simplification but **viewed with disdain** for **obfuscating results** and creating **syntax issues**

Instructors' pedagogical philosophies often did not correlate with student outcomes from exam problems in either representation.

No detectable correlations between **instructional** style and...

...students' choice of **numeric strategy.**

...frequency of **blank submissions.**

Proportional odds ordinal logistic model:

$$Y|R = \text{Num.} \sim I + M + IM$$

$$Y|R = \text{Sym.} \sim I$$

Y: problem score
R: problem representation
I: instructional style
M: student method

- **No factor** was a significant **predictor** of score for **more than 2/5 parts** in either model.

- **When instructional** style produced a significant **impact,** students receiving **agnostic** instruction often scored **highest** while **symbol-leaning** instruction scored **lowest.**

More work is needed to understand if instructional interventions can narrow the gap between numeric and symbolic understanding.

- Differences in exam scores between representations **cannot be wholly attributed to when students plug in numbers.**
- Our instructors each adopted one of **four distinct pedagogies** when addressing representations in introductory mechanics.
- Differences in **instructional styles did not affect** students' choice of problem-solving **method** and left **inconclusive** impacts on problem-solving **proficiency.**

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General proportional odds ordinal logistic model:

$$\boxed{\ln\left(\frac{P[Y(X) \geq k]}{P[Y(X) < k]}\right) = \alpha_k + \beta X}$$

$$\frac{P(Y \geq k)}{P(Y < k)} = \frac{p}{1-p} \equiv \mathrm{Odds}$$

$$X \in \{0,1\} \Rightarrow \frac{(\mathrm{Odds}|X=1)}{(\mathrm{Odds}|X=0)} = e^{\beta}$$

Y: measured ordinal outcome
X: categorical predictor variable
P: probability of…
k: index; same possible values as Y w/o min
α_k: fitted thresholds
β: fitted effect of predictor variable

Proportional odds assumption: $\beta_k = \beta$

Our attempted model:

$$(\mathrm{LogOdds}|R=0,I,M) = \alpha_{k,0} + \beta_1 I + \beta_2 M + \beta_3 IM$$
$$(\mathrm{LogOdds}|R=1,I) = \alpha_{k,1} + \beta_4 I$$

$$R = \begin{cases} \text{0, Numeric} \\ \text{1, Symbolic} \end{cases} \qquad I = \begin{cases} \text{0, Symbol-centric} \\ \text{1, Symbol-leaning} \\ \text{2, Representation agnostic} \\ \text{3, Number-centric} \end{cases} \qquad (M|R=0) = \begin{cases} \text{0, Expert-like} \\ \text{1, Intermediate} \\ \text{2, Novice-like} \end{cases}$$

- Equilibrium (part 1):
  - Frictionless ramp
  - Unknown friction between boxes
  - Unknown force on lower box
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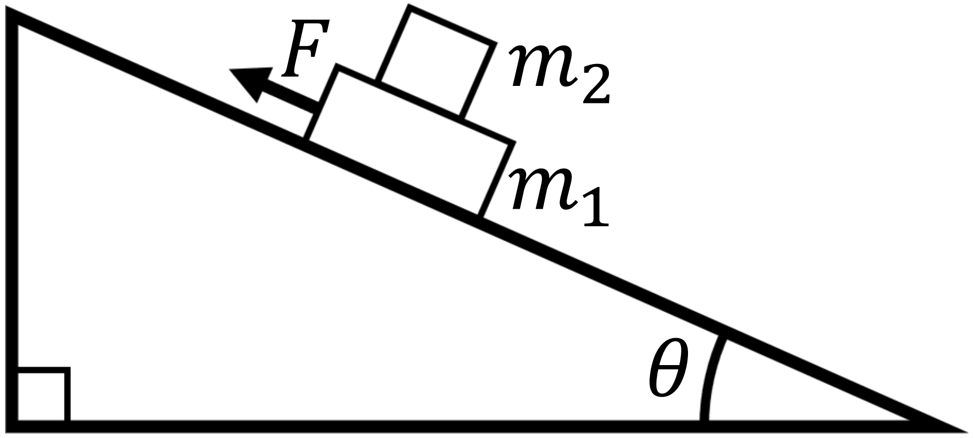
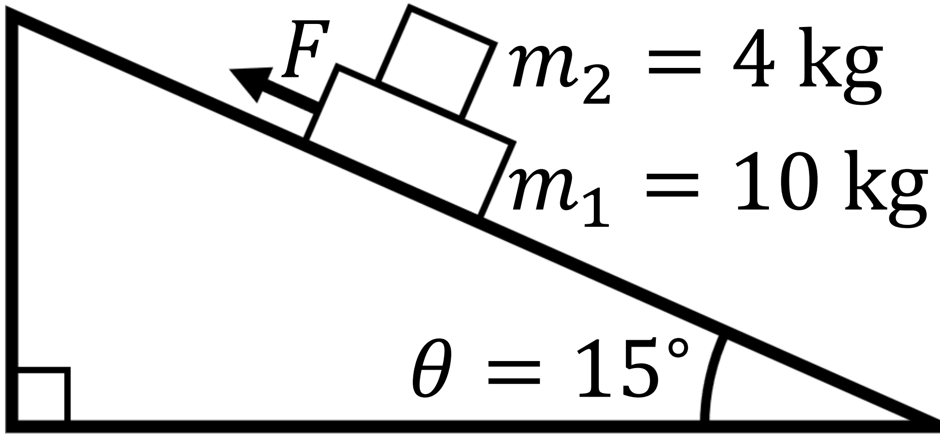
  - **Find μ_k between boxes**

- Equilibrium (part 2):
  - **Find magnitude of F**

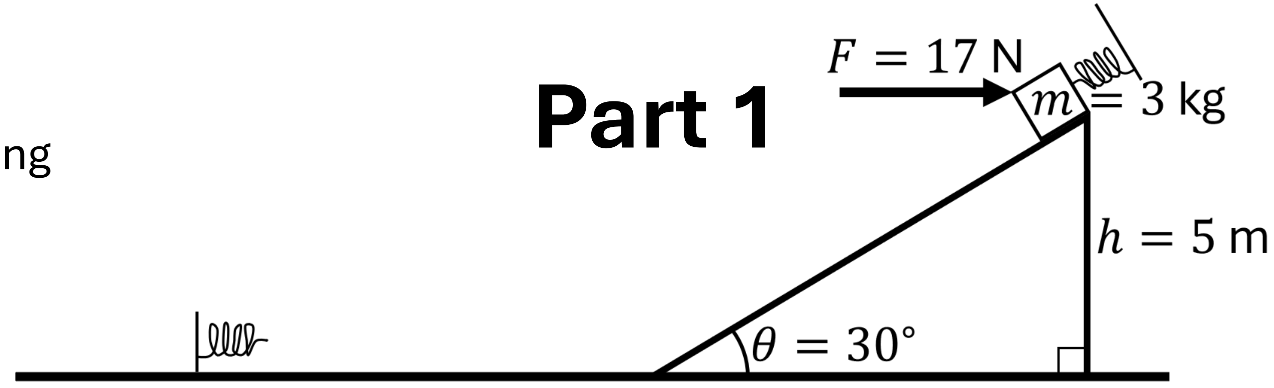
Symbolic



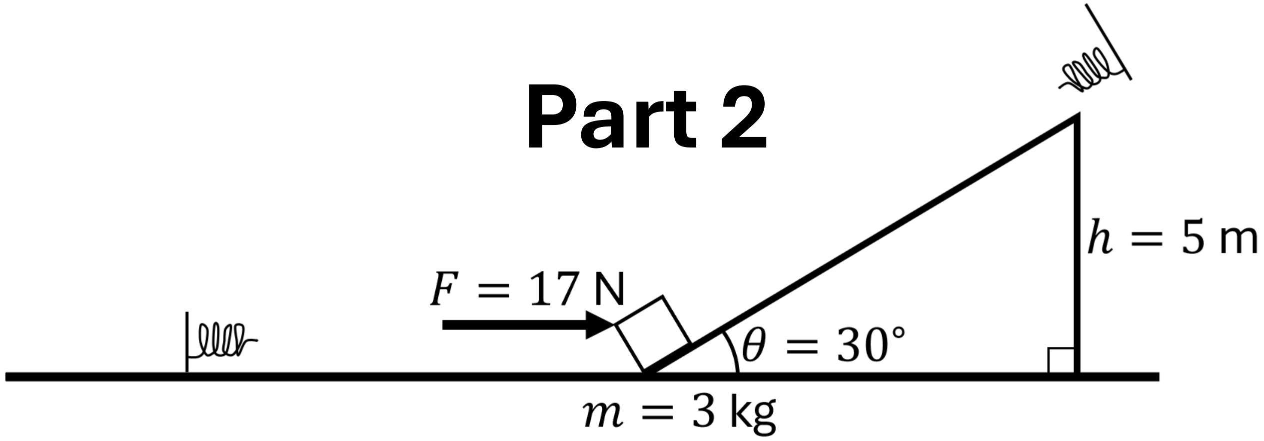
Numeric



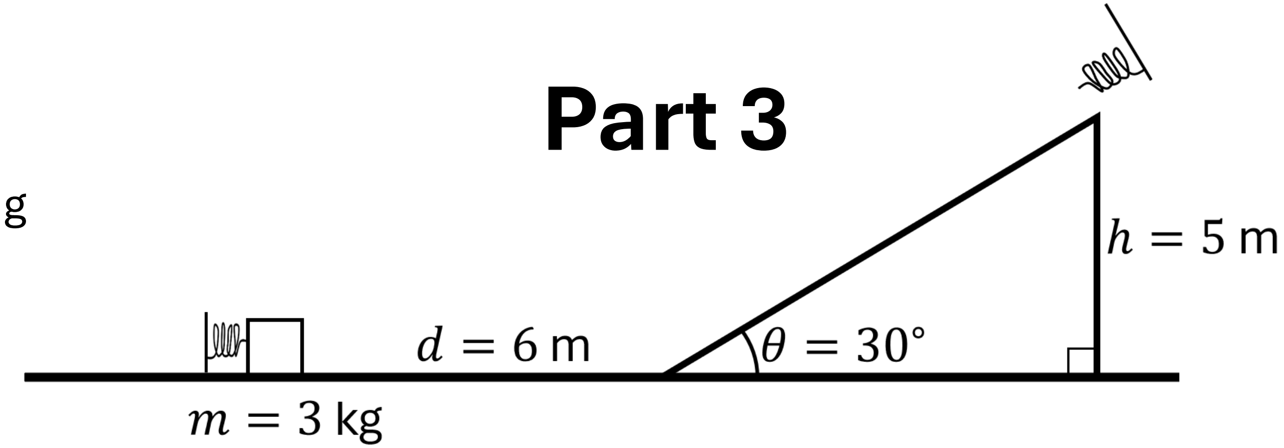
- Energy (part 1):
    - Initial: box held motionless against compressed spring
    - Released box slides downward
    - Constant horizontal force along length of ramp
    - **Find work by F on box along whole ramp**



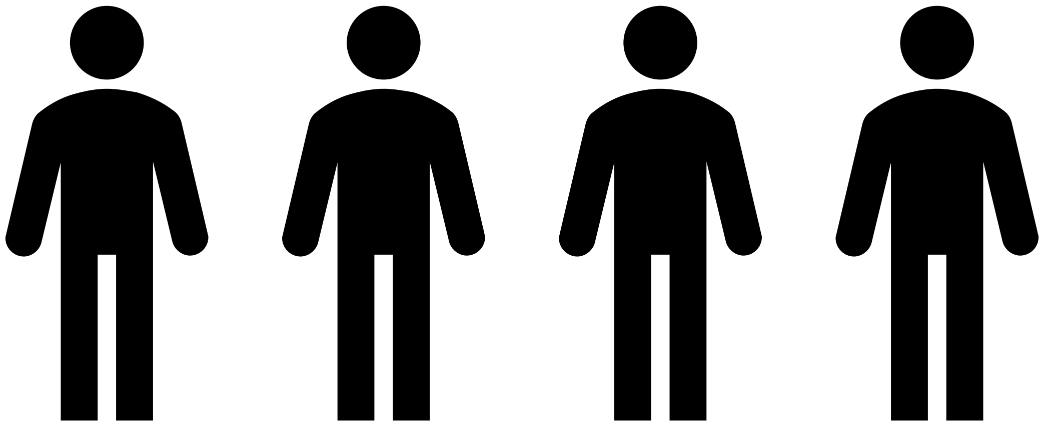
- Energy (part 2):
    - **Find kinetic energy of box at bottom of ramp**



- Energy (part 3):
    - Frictionless horizontal surface at bottom of ramp
    - After ramp, box continues until colliding with a spring
    - Spring compresses until box comes to a stop
    - **Find length by which the spring compresses**



Symbol-centric

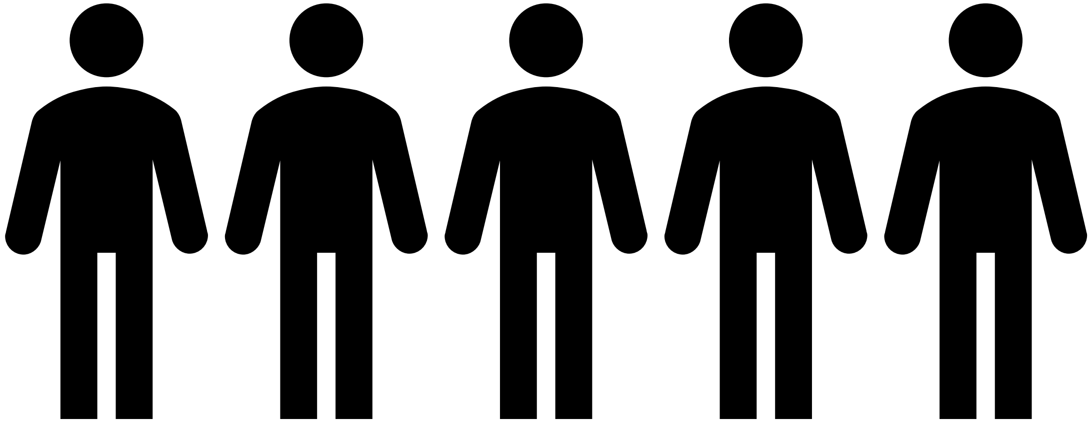


When leading class…

- Heavy **emphasis on symbolic** mastery
- **Numbers used sparingly** and cautiously with **highly specific goals** (usually to simplify unwieldy algebra)

- "I generally **hate numbers**."
- "**Numbers can be a crutch** for the student who is not so adept. **Symbols force them** maybe to **think more carefully** or at least will **expose issues** more readily."
- "There's **occasionally** a problem where if you don't use numbers, the algebra becomes so awful that you'd be **better off plugging in.**"
- "What's unwieldy to them is **not unwieldy from my perspective.**"

Symbol-leaning

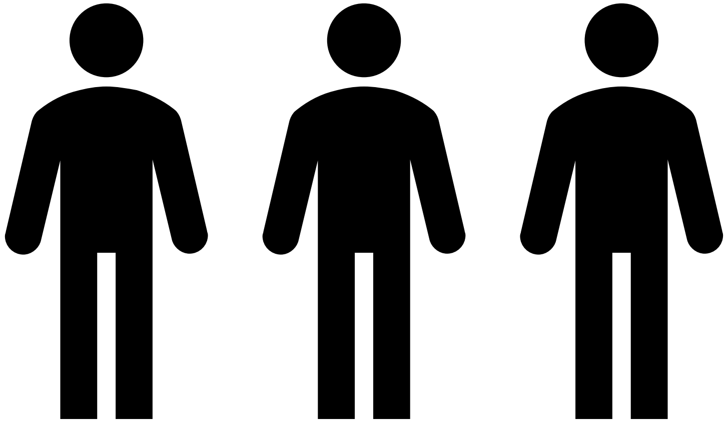


When leading class…

- **Balanced exposure** to numbers and symbols…
- …but **symbolic** mastery remains **underscored during numeric** exercises

- "**When I show examples**, some that are symbolic, some that are numeric, I mostly do it to **give them practice in both ways**.
- "**Even though it's a numerical problem,** usually when I show them how to do it…, **I really recommend** doing all the steps all the way to the bottom with **symbols**."
- "I always tell them **a problem can be there without the numbers**. So, you need to be familiar with the concepts, and it's going to be the same for numerical values and symbols. **There's nothing different.**"

Representation Agnostic



When leading class…

- **Balanced exposure** to numbers and symbols due to **representational agnosticism**

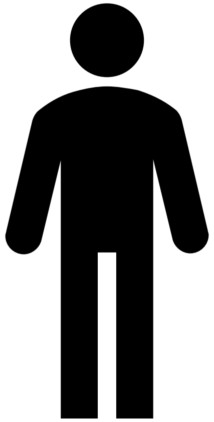
- Focus on universal problem-solving skills and **fluency across all representations**

- "**They should know both.** They should know how to handle units. They should know how to use expressions. They should know how to do the symbolic algebra. **I don't see that as sort of a big decision.**"

- "**Representations are super important,** and **not necessarily numbers versus symbols**, but, for example, representing problems graphically, drawing sketches, trying to represent vector quantities symbolically, trying to do the mathematics graphically. This is all built on the notion of **developing the conceptual picture** before running to grabbing equations…."

- "I guess we tell them, just in case, to avoid numbers. When they're solving the homework or the exams, I think it's **whatever the option they want, I think it doesn't really matter.**"

Number-centric



When leading class…

- **Preference for numeric**, real-world understanding

- **Symbols** used for simplification but **viewed with disdain** for **obfuscating results** and creating **syntax issues**

- "It's good for them to make contact with the actual numbers. If you just think about [universal gravitation] from the formula…, what if I have two basketballs? …**If you don't run the numbers on that, you don't realize** that they basically don't [attract]."

- "There are these letters that somehow, **we're supposed to pretend we know** what they are, because we don't actually know what the numbers are. But we're pretending that we do, and so we need to take one of the unknown letters and rearrange it to get it in terms of the letters that we've decided that we know. And sometimes, **I worry that we just lose students who understand the problem anyway**…."

- "**A symbolic expression times another** symbolic expression **doubles in length** of the symbolic expression."

- "**With multiple choice**…, students… would just say, 'I picked that one because **all the other answers looked wrong.**'"